THE USE OF MATHEMATICAL MODELS IN ALFALFA PEST MANAGEMENT

A. P. Gutierrez
Associate Professor of Entomology
University of California, Davis

The alfalfa ecosystem is biologically rich and quite complex. While we can consider it an ecosystem, it is not allowed to evolve, rather it is managed for profit. It can be easily disrupted by man when the dynamics of various herbivores conflict with his economic interests. The activities of herbivorous pests like the exotic Egyptian alfalfa weevil (Hypera brunneipennis (Boh.) = EAW) is just such a case. The EAW problem has become so severe in California as to require routine pesticide applications for economic cultivation of alfalfa.

The problem is complex, but agriculturists usually seek simple solutions; ignoring, for many reasons, the many trade-offs which are present in the system. For example, the farmer might spray when the cost of spraying is more than the benefit. Hence, the questions farmers really want to answer are not only whether to spray or not, but also when and how much. There is, in fact, an optimum with regards to both variables. Commonly, the spraying induces pests which might not normally be pests (i.e. secondary pest outbreaks) or in longer perspective, the treatments may engender the development of pesticide resistance. These problems clearly cost farmers money. Furthermore, the pesticide policy adopted by farmers may conflict with the policy that is optimal for society which wishes to prevent undue pollution, contamination of foods and other environmental costs. These are sophisticated questions, and they require sophisticated analyses. Classical statistical analyses have failed to be robust enought to answer these questions, hence many workers have sought to construct mathematical models of these complex systems.

What are mathematical models? Most people who construct them (systems analysts of one stripe or another) would consider them to be reduced but concised descriptions of the real world. Because the real world is very complex, the human mind seeks aids to enable it to grasp the essence of the problem, generalize it and lastly expand this knowledge and perception (i.e. models). This method of problem solving has now surfaced in pest control, and this paper reports in a general way the analysis of the EAW/alfalfa interaction in California.

The Biology of Alfalfa

Alfalfa is grown both as a forage and seed crop. During late fall, frosts kill the aerial parts of the plant, which begin regrowth when conditions become favorable again during late winter or early spring. The crop grows approximately 900 to 1000 D^o (>42^oF) at which time it is cut (approximately 1/10 bloom). Regrowth of the crop begins from the root reserves, and continues until its photosynthetic machinery can provide all of the growth requirements. Alfalfa in California is cut several times during the season.

The Plant Model

Figure 1 depicts the flow of plant dry matter. The size of the tank varies with the size of the plant, as do the various influxes (new photosynthate) and effluxes (dry matter growth (W) and respiration (θ)). The amount of photosynthetic material (P) which is produced at time t.

$$= f\text{ (solar radiation, leaf surface, age, } \dot{H}_2O,$$

In the simplest model the total plant weight (w) is composed of plant tissues (Q) and carbohydrate reserves (C)

$$W = Q + C \qquad (1)$$

The growth rate of the crop is

$$dW/dt = dP/dt - \dot{\theta}_1 \, dQ\text{-}dt - \dot{\theta}_2 \int Wda - \dot{\theta}_3 \, dP/dt \qquad (2)$$

Note that $\Delta t = \Delta a$ in the model, but this is not a necessary condition. The various θ_i are respiratory losses associated with converting P to Q, maintenance respiration, and production of P respectively. Other θ_i could be assigned, but the above are the dominant ones. If we differentiate (1) with respect to t and solve 1 and 2 simultaneously for dC/dt, we get

$$dC/dt = \{dP/dt - (1+\dot{\theta}_1)\cdot dQ/dt + \dot{\theta}_2 \cdot \int Wda \,] \,/(1+\dot{\theta}_3) \qquad (3)$$

or the rate of new carbohydrate accumulation in plant reserves. A similar mass flow model was presented by Jones et al. (1974), but that model lacked age structure for W.

In our model, the growth of the crop depends upon the supply (s) of available carbohydrate $(P(t) + \alpha \cdot C)$, where α is the fraction of C available for growth at time t. This supply is counter balanced against the demands (D) for maximum growth

$$(D_t = [\dot{\theta}_1 dQ/dt + \dot{\theta}_2 \int Wda + \dot{\theta}_3 dP/dt + \dot{L}(t) + \dot{RT}(t) + \dot{ST}(t)] \cdot \Delta t =$$

$$[\theta_1 + \theta_2 + \theta_3 + \Delta L + \Delta RT + \Delta ST])$$

If S>D, then C(a=o) = S-D, but if S<D then the plant grows only a fraction of its potential, and the following priority scheme is operative

$$r_1 = (S - \sum^{3} \theta)/(\Delta L + \Delta ST + \Delta RT)$$

and

$$dW/dt = \frac{L}{t} + \frac{dRT}{dt} + \frac{dST}{dt}) \cdot r$$

The population model for the alfalfa crop consists of three models

$$\frac{\partial L}{\partial t} + \frac{\partial L}{\partial a} = -\mu(\cdot)\cdot L(t,a)$$

$$\frac{\partial RT}{\partial t} + \frac{\partial RT}{\partial a} = -\mu(\cdot)\cdot RT(t,a)$$

$$\frac{\partial ST}{\partial t} + \frac{\partial ST}{\partial a} = -\mu(\cdot)\cdot ST(t,a)$$

which are coupled via the metabolic pool model (r = S/D) (see above and (Wang, Gutierrez and Oster, In Press). Both t and a are measured in physiological time since the last cutting. The models are the continuous form of the well known Leslie Matrix (Leslie 1945). In fact,

$$\mu(\cdot) = f(r, \text{herbivore damage, frost, cutting cycles} \cdots\cdots)$$

Herbivore Activity

Egyptian alfalfa weevil adults and larvae attack the growing tips of the alfalfa stems, and consumes leaves (it prefers the younger aged leaves). This activity alters the age structure of leaves and the growth form of the plant; both of which affect photosynthate production. In addition, its feeding causes the plant to divert photosynthate from growth to wound healing.

A complete model for the pest (see Gutierrez, Christensen, et al., 1976) would describe not only the feeding activities, but also the dynamics of the weevil population. Let us assume, for simplicity, that we have some time pattern and age structure of weevils which are attacking the crop. The mass of the crop at time t under a given weather regime will be influenced by the feeding it has sustained prior to that time. The dry matter required for wound healing ($H = b \cdot E$) would be a further net loss proportional (b) to the amount eaten ($E_t = \int F(a) \cdot N \, da$) by the population N, where F is some age dependent consumption rate. Eqn. 2 then becomes

$$\frac{dW}{dt} = \frac{dP}{dt} - \dot{\theta}_1 \, dQ/dt - \dot{\theta}_2 \int^{m} \ell x(a) \cdot W \, da - \dot{\theta}_3 \frac{dP}{dt} - \frac{dH}{\ }$$

and $\ell x(a)$ is the survivorship value from weevil feeding on leaves ages o to m. If this equation is substituted for 2 and used to obtain 3, we can see that H affects dC/dt directly. This fact becomes especially important if we consider that the crop's regrowth occurs from the reserves. Hence we see that severe defoliation can affect not only the current crop, but also the next one via the reserves and/or the destruction of whole plants. The latter aspect has not been completely studied. Whether the farmer likes it or not, this is the system which he must manage.

Economic Aspects

The agricultural practices employed by farmers are designed to achieve high and consistent yields. The application of pesticides is one of these practices and, as is well-known, has both beneficial and harmful side effects. These side effects are called externalities. While the farmer's decisions may affect society at large, society under our present competitive system has little effect on the farmer's decisions. Regev, Gutierrez and Feder 1976 and Regev (In Press) discuss these conflicts in greater detail.

In general, individual farmers wish to maximize profits ($\pi = [B(x) - \emptyset(x)]$), which in the case of pest control equals the benefits (B) of the pesticide applications minus the costs ($\emptyset$).

Figure 2a depicts the relationship between increasing pesticide use and the resulting benefits and costs in dollars. The important point here is that increasing pesticide use (i. e. a linear cost) does not result in proportionately the same increasing trend in benefits. Simply put, Figure 2b shows that the optimal amount of pesticides to use ($\hat{X}$) occurs when $\frac{dB}{dx} = \frac{d\emptyset}{dx}$. The value of $\hat{X}$ is not easily obtained in practice, as it varies according to pest density, the value of the crop, weather and many other considerations which may be known or unknown by the farmer. In fact, the farmer is most likely not able to accurately assess this, and hence experience and economic success provide the necessary rules of thumb; which, in practice, may be far from optimal.

It is possible that using sophisticated mathematical techniques, we can incorporate much of the complexity we observe in nature and arrive at an estimate of $\hat{X}$. The model must incorporate detailed information about the plant's growth responses to weather, the pest's biology, its age dependent damage rate and its impact on the plant. The effects of timing and quantity of pesticide use on secondary pest outbreaks can also be incorporated. Such models have been developed (Regev et al.,(1976) and Regev, Shalit and Gutierrez (In Preparation)).

Some very practical results can accrue from this kind of work. For example, the model tells us the optimal time and quantity of pesticides to apply. In the case of the Egyptian alfalfa weevil in California, Regev et al., (1976) found that the pesticide applications should be made during the winter period after all of the adult weevils have returned to the field, and before large numbers of eggs have been deposited (Figure 3). There is a problem with this recommendation, because it presupposes that we know the weevil population density, its damage potential and that consistently effective pesticides for use during the winter period exist. But the model has further use in addition to solving merely practical problems. As we have pointed out, the actions of the farmer may be at odds with those of society. Regev (In Press) incorporates these costs (ξ) as some penalty to the farmer as follows,

$$B(x) - \phi(x) - \xi(x)$$

and shows that the optimal amount of $\hat{X}$ to use from the societies' point of view may vary radically from what is optimal from the farmer's point of view. Figure 4 shows that the optimal value for X may be lower, but this doesn't necessarily mean that farmer profits are lower. This notion of environmental responsibility is the focal point of the current clash between environmentalists and the pesticide user. In general, if the farmer had to pay the environmental costs, he would likely use less pesticides. Like all generalities, it doesn't hold for all cases.

References

Gutierrez, A. P., J. B. Christensen, C. M. Merritt, W. B. Loew, C. G. Summers and W. K Cothran (1976). Alfalfa and the Egyptian Alfalfa weevil (Coleoptera; Curculionidae). Can. Ent. 108: 635-648.

Jones, J. W., A. C. Thompson and J. D. Hesketh (1974). Analysis of SIMCOT: Nitrogen and growth. Proc. Beltwide Cotton Prod. Conf., Memphis, pp. 111-117.

Leslie, P. H. (1945). On the use of matrices in certain population mathematics. Biometrika 35:213-45.

Regev, U. (1976). Economic conflicts in plant protection; individual versus social decision. Invitation talk, XXV Anniversary of the European Plant Protection Organization, Paris (Oct.)

Regev, U. A. P. Gutierrez and G. Feder (1976). Pests as a common property resource: A case study of alfalfa weevil control: Amer J. Agric. Econ. 58: 186-192.

Regev, U., H. Shalit and A. P. Gutierrez (In Progress). Economic implications of pest resistance: theory and application.

Wang, Y., A. P. Gutierrez and G. Oster (In Press). A general model for plant growth and development: Coupling plant herbivore interactions. Can. Ent.

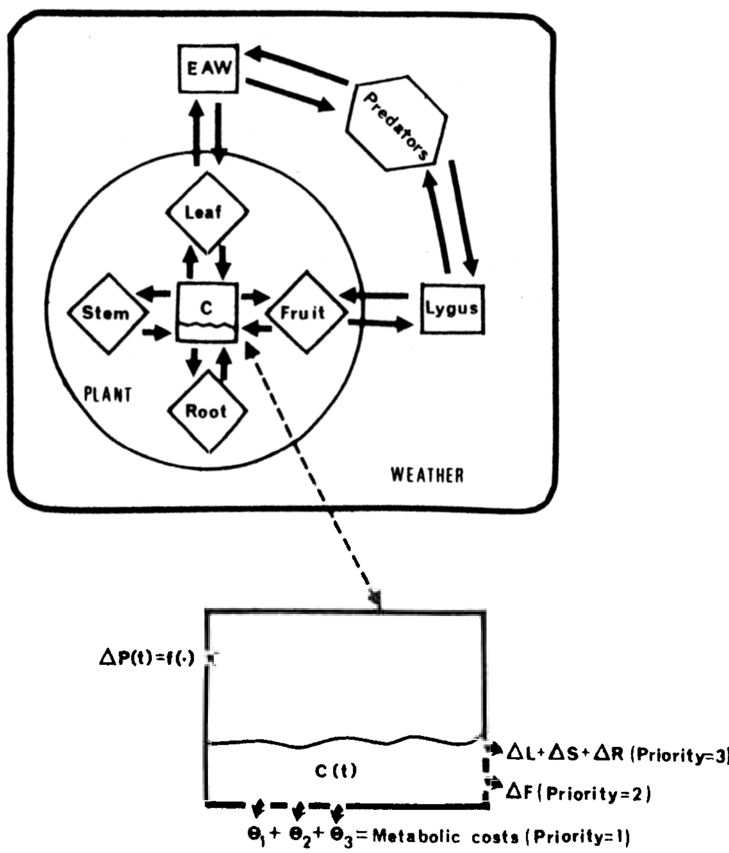

Figure A simplified carbohydrate pool model for allocating mass to various plant tissues. ΔP is the new photosynthate, Cda is the carbohydrate reserves, the θ_i are metabolic costs, ΔR, ΔS, ΔL and ΔF are increments of maximum growth potential for root, stem, leaf and fruit respectively, while the $0 \leq r_i \leq 1$ are supply-demand ratios.

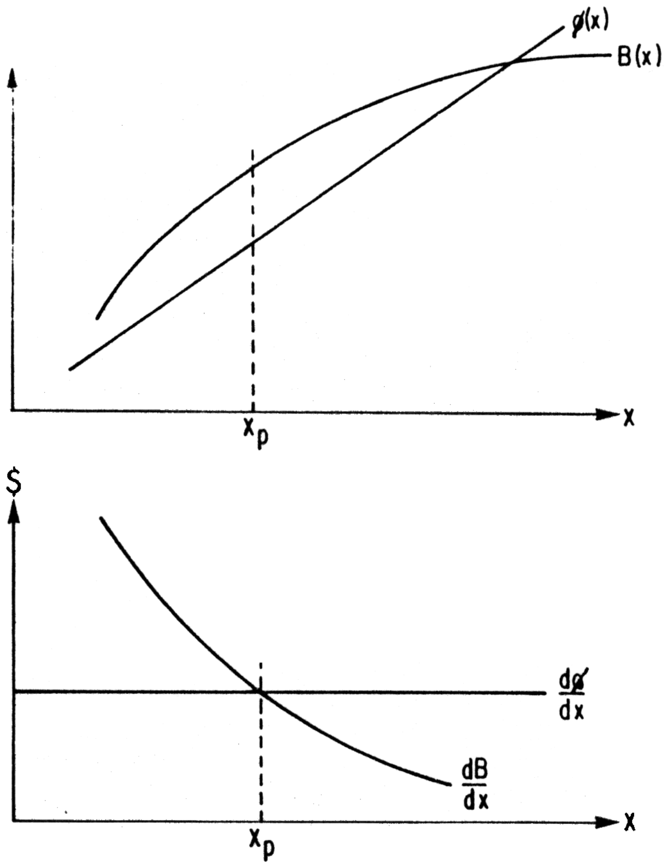

Figure 2 A comparison of benefits (B) and costs (Ø) of using increasing amounts of pesticides. (A) The observed responses and (b) the derivatives of those functions. The inter-section of the two lines in (b) is the optimal policy (c.f. Regev 1976).

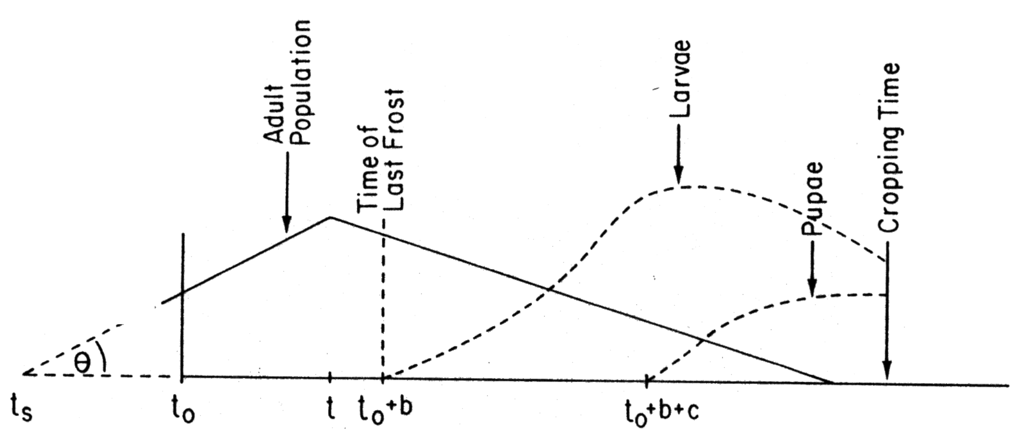

Figure 3 The phenology of the Egyptian alfalfa weevil. θ is the infestation rate, t_s is the beginning of adult emergence, t_o is the time when egg laying begins, t^* is peak adult population, t_o+b is the time of last frost and the appearance of the first surviving larvae, t_o+b+c is the time of first pupa and t_f is harvest time for the alfalfa. The optimal time for control is t^* to t_o+b.

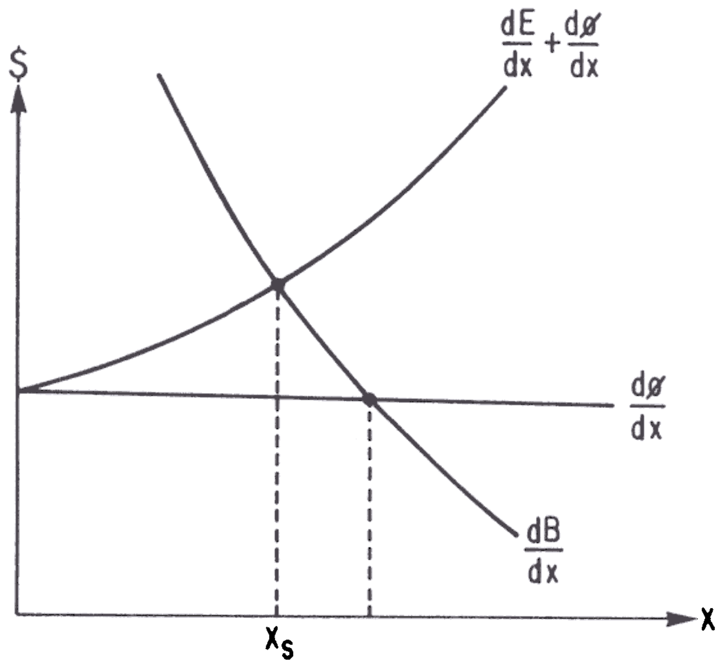

Figure 4. The incorporation of environmental costs in the optimization function (c.f. Regev 1976).